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Photo: Martyn Farr

Cave River Level Telemetry at Dan-yr-Ogof

Ice Cave Photography using a Smartphone, Turning 3D Models into Cave Maps, LORAN Developments, Cave Radio Antennas and Reciprocity, Touchscreens

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Front Cover:
The main river in Dan-yr-Ogof. After sufficiently heavy rainfall these cascades become impassable and the low canal beyond fills to the roof. This is close to Pot Sump where Stuart France is tracking the water level continuously, as described in his article on Page 16 of this issue.

Photo: *Martyn Farr*

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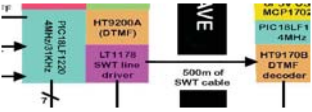

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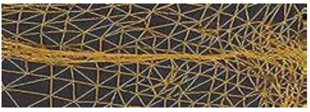

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Ice Cave Photography by Haukur Ingi Einarsson of Glacial Adventure in Iceland

Cave River Level Telemetry at Dan-yr-Ogof

__Stuart France__ describes a system that combines two telephone technologies to measure and transmit regular cave river water level readings over a long, fixed single-wire telephone cable to surface data logging equipment and subsequently to the cloud.

Introduction

This is the first of two articles covering the acquisition of water depth data from far underground, its transmission to the cave entrance where it is currently saved, and setting out some options for making experimental results available in near real-time on the Web. The viability of encoding data using Dual-Tone Multi-Frequency (DTMF) sent serially over a Single-Wire Telephone (SWT) circuit at audio level to a surface decoder has been proved.

DTMF is the dialling system used by touch-tone telephones, best known to users as the discordant beeps heard when pressing a number key. SWT field telephones have been used for many years with considerable success, reliability and ease of use in cave rescue incidents and on expeditions. They operate over kilometres of just a single wire because the return signal goes through the ground.

The follow-up article will explain how an Internet connection and a webpage were added so as to provide cavers with access to the current river level and recent history on smartphones, laptops etc.

Operational Context

Dan-yr-Ogof (DYO) is a popular destination for cavers in South Wales with 15 km of wild passages [**1**] lying beyond the managed Show Cave area, which has concrete pathways and steps to facilitate tourist tours [**2**]. Cavers meet with passages both very large and small, wet and dry, mainly water-washed and mud-free with very pretty formations. There is potential for significant discoveries to be made within DYO's large water catchment area.

DYO has two main river sinks, but the water resurging at the Show Cave is mainly due to percolation through the mountain surface, minor streams, etc. There are two rivers within the cave which join and share a resurgence. These have different flooding rates and earlier work [**3**] has shown, not unsurprisingly, that the underground river associated with the closer sink floods first, then, as its waters recede, the flow from the distant sink creates a secondary and larger flood pulse observed at the cave resurgence.

About 400 m inward from the cave entrance is Lake 3, the largest and deepest of four lakes, fed by a very active river with, at times, powerful cascades. During a drought, one can stand on bedrock ledges in an airspace with a minimum height of two metres above the active river. The river, during a flood event and by then impassable, sumps in a canal section (seen in the background of the front cover photo) thus cutting off any party of intrepid cavers inside the wild cave until its level falls – which can take over a day. People have become entrapped, plus some near misses. Given that most of the river water is due to percolation, the rainfall history over the past several days or weeks is relevant, as is the weather forecast, the current river level, and any rising or falling trend not only over the past 24 hours, but also for the past 1 - 2 weeks since floods can build incrementally. Unfortunately, none of that data is available, let alone readily on a website where caver safety can be put into a proper context before deciding to explore in the wild cave.

Lake 3 is connected by a permanently flooded, yet easily diveable tunnel, emerging underwater in a large pothole with static water called Pot Sump. Its surface level tracks that of the active river at the canal because it is a U-tube. Pot Sump was identified more than a decade ago as the ideal place to put a water depth sensor to collect cave hydrology data. But nothing was done because the caving advisory committee then had to approve the project. These people were cavers who lacked relevant experience, but that did not stop them from mandating a method that was doomed to failure. It has been a long wait for a new more open-minded caver committee that is willing to accept engineering advice.

Throughout all this time, indeed as long as I can remember, the Show Cave owners have been very supportive of caving activities and explorer access, with a

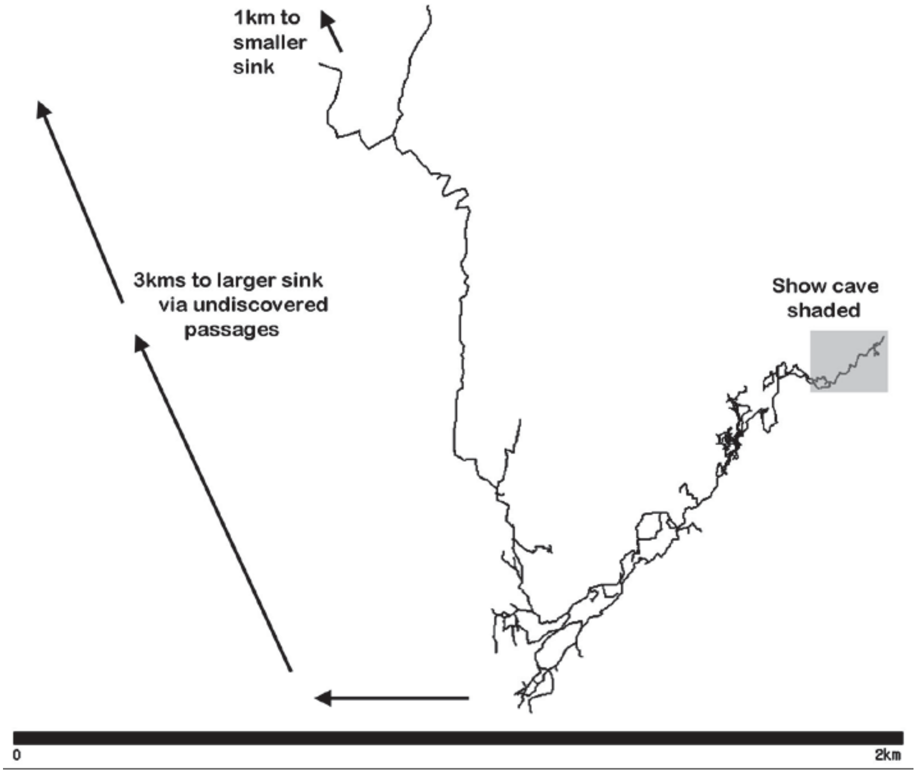


Figure 1 – Dan-yr-Ogof 2km Scale Survey

watchful eye on cave conservation, and always with the desire for input from the caving community. So, I would like to thank them here for building and supporting this very special and enduring relationship.

Submersible Depth Gauge Sensors

The submersible depth sensor module now in use at DYO is the same type that the author fitted a decade ago in Ogof Ffynnon Ddu 1 (OFD1) on the other side of the same valley. Both have 4 m water depth as full scale. Many years of interesting data have been collected in OFD1, initially on my data loggers [**4**]. In recent years, a second identical module has been installed by South Wales Caving Club (SWCC) nearby which, via a series of cable and radio segments, sends live data back to their cottages about 1 km away. This second sensor (or its comms) has sometimes been unreliable, so the original sensor was then connected into the cable-radio system which rendered it unreliable as well, so the data can only be used when both live readings agree, there being no longer any data logger near the sensors. The issue is ongoing.

The sensors are Druck (General Electric) PTX1730 units. That type has now been superseded by PTX1800 and Unik5000, costing about £300-400 each depending on the options required. As it happens, another second-hand PTX1730 sensor with a depth range of 0-4 m H_2O with a 6 m bonded-on cable became available to me at no cost from a business contact, so this was gratefully accepted for the DYO project.

Various depth sensor outputs are available, but 4-20 mA is a good choice because the sensor then conducts 4 mA for no depth of water and 20 mA for its full scale. The sensor has only two wires (its power supply) which is very convenient because the resistance of the power line and the DC voltage at the sensor is irrelevant within reasonable bounds. The sensor current is passed through a sensing resistor, grounded on one side, so as to obtain a voltage relative to ground which is linear with water depth. This voltage is then digitised as the sensor reading and stored by a local data logger or transmitted elsewhere.

The 4 mA current offset might seem a nuisance, but it is a way of distinguishing a "no depth" reading from a fault. The first OFD1 logger used a 12-bit MCP3201 ADC chip connected to a PIC16LF84 processor while the later DYO system uses the 10-bit ADC module within a PIC18LF1220. This ADC has optional Vref- and Vref+ inputs, where the former pin can be used to cancel the voltage arising from the 4 mA offset. The formula of a 186 Ω current-sensing resistor and a 10-bit ADC running at 5.0 V with the sensor offset cancelled scales the output to a convenient 5mm per AD unit. PIC18s have power-saving options such as clock speed switching, which enables the underground sensor system to be a microwatt device in terms of average power. We expect it will run for a year using AA alkaline batteries and not require regular maintenance visits wearing a wetsuit and wellies.

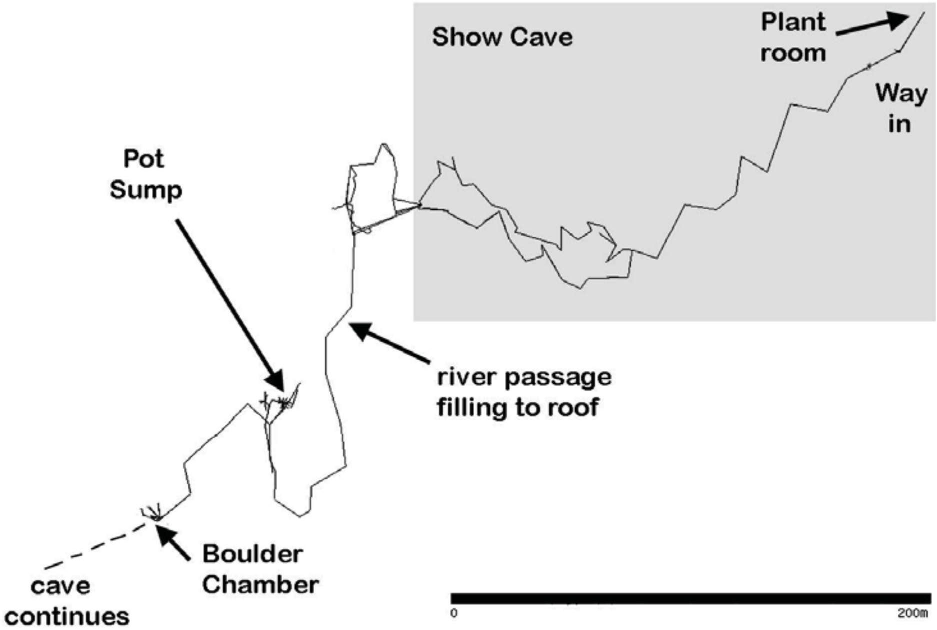


Figure 2 – Survey Detail

The water level sensor was fitted at Pot Sump.

The second-hand sensor acquired for DYO was found to have a 6mA offset (not 4 mA) but it then responded linearly to depth in the expected way. Linearity was evidenced initially by a calibration exercise indoors using a one-metre-high column of water in an end-capped drainpipe in a bucket to catch the slops, see *Figure 3*.

Figure 3 – The Indoor Calibration Rig

This was later repeated outdoors, but using a 4m-long piece of sink drainpipe at Pwll-y-Cwm in the Clydach Gorge, which is a deep flooded pothole, see *Figure 4*. The fallen tree was an unexpected bonus as it prevented the author from slipping into the pothole or the river. The pothole, in fair weather, contains calm water within its vertical sides while the active river is kept back behind a strong stone wall built by cavers. This experimental setup tested the system at depths up to 3.5 m in 10 cm steps.

Telephone Cables in DYO

An existing single-wire telephone (SWT) cable laid for cave rescue purposes has existed for decades. It runs from Boulder Chamber, a dry and roomy area well beyond the active river, to the plant room at the Show Cave entrance. This cable was extended underground by 100 m to Pot Sump and successfully tested by this project: first with speech using the standard cave rescue phone handsets, then later with DTMF tones representing a series of fake water depth readings. The cave rescue team benefits from this project

Figure 4 – The Outdoor Calibration Site at Pwll-y-Cwm

Showing the 4m length of drainpipe which was submerged vertically during the calibration procedure.

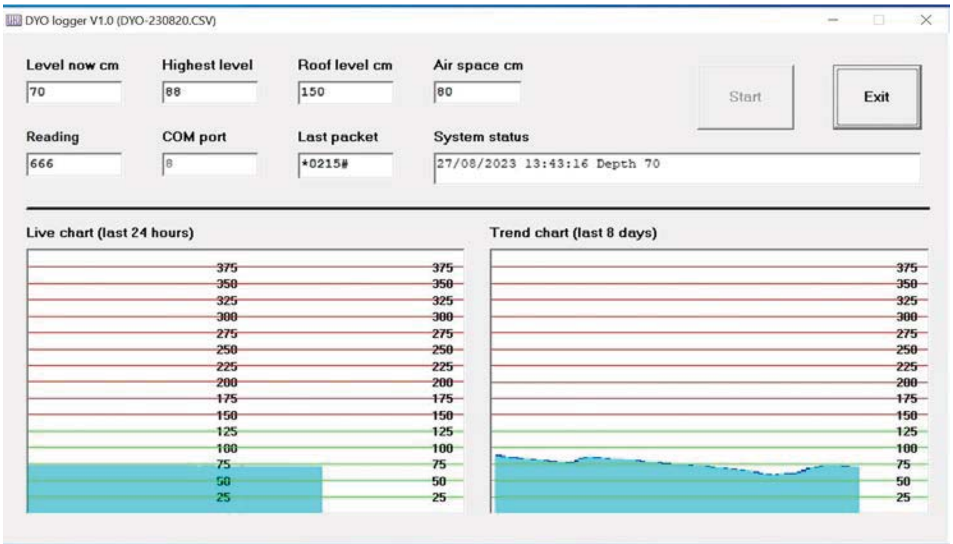
because their emergency phone line is continuously tested by sending water depth data across it.

The Show Cave business has modern mains electrics for lighting, audio and other uses. Inevitably, there will be some mains interference on the SWT line which manifests as background buzzing. Our solution is DTMF encoding, explained below, so the water depth readings are sent as audio. The audio circuit is similar to that in the SWT handsets, which the author developed and built for participating UK teams during 2013 [**5**]. In effect, the water depth sensor obtains a reading and then 'dials' a fake phone number over the phone line which represents the water depth in AD units. The 'dialled' number is decoded by a DTMF receiver near the cave entrance, which forwards ASCII data over a serial-to-USB converter into a laptop PC running on mains power in the Plant room.

Initially, a HyperTerminal program acquired the data, but later that was replaced with a custom Windows app to display two moving charts of the recent river depth history. These show 15-minute readings for the past 24 hours and 2-hourly readings for the past week, see *Figure 5*. Local rainfall data will be acquired and included in a future system enhancement. The final step will be the cloud connection to make recent results available to everyone.

DTMF Dialling

DTMF, introduced from 1963, is the international standard for keying numbers on modern touch-tone telephones. DTMF has a 4-bit code representing the digits 0-9, the hash and star symbols as seen on telephone keypads, plus the letters A-D which are not implemented on consumer phones. So, a reading of 1023 decimal could be dialled out on the phone line as DTMF tone 1, tone 0, tone 2, and tone 3. However, it is easier to work in octal until the last moment. Thus, the water depth reading is sent as a 4-digit octal number bracketed by star and asterisk characters where *1777# represents full scale since the ADC is 10-bit. The receiver passes the ASCII character representation of the entire packet to the laptop over USB, i.e. "*", "1", "7", etc. There is no timestamp within the packet because the PC knows the current date-time when it receives each packet.



DYO logger V1.0 (DYO-230820.CSV)

Level now cm	Highest level	Roof level cm	Air space cm
70	88	150	80

Reading	COM port	Last packet	System status
666	8	*0215#	27/08/2023 13:43:16 Depth 70

Figure 5 – Windows App Display

Moving charts show recent readings for the past 24 hours and over the past week.
(Showing data from the first week of operation.)

The DTMF chipset comprises a Holtek HT9200A generator and HT9170B decoder. These parts are available on eBay. The datasheet example circuits were followed, and they worked, but the example algorithms shown in a kind of assembly language seemed unlikely to work without additional code, as in our PIC firmware, to factor real-time issues such as signal propagation delays.

Transmitter Design

Cave rescue phones drive the SWT line at 12 V, but this was changed to 3.3 V in the DYO sensor system so as to be compatible with typical modules that the receiver might use to reach the Internet without involving a PC. The commonplace LM258 dual op-amp used in the rescue phones was changed to LT1178 to save power (a micropower precision op-amp which I had in plenty). The final firmware version for the SWT transmitter runs at different clock speeds at different times to save power, it goes into sleep between readings, and it only turns on peripherals when needed, so the average quiescent current is below 50 µA.

The transmitter provides for a plug-in 16×1 character LCD panel (search HD44780 on eBay) to display the current depth reading continuously while connected, which is useful for calibrating the Druck sensor, such as during my outdoors calibration at Pwll-y-Cwm. These displays require 8-bit parallel data input plus clock and a logic pin to indicate if the incoming byte is a display command or a display character.

My LCD plug-in box has been used in many projects. It does serial communications over just four I/O lines: these clock 8 data bits into a 4015 shift register; set the LCD logic for command or character mode; then clock the module to make it read 8 bits in parallel from the

Figure 6 – Boulder Chamber

The rescue phone cable termination box and the extension wire to Pot Sump being reeled out.

4015. The LCD needs a 5.0 V supply, but my circuits often run from 3×AA batteries which provide 3.0-4.8 V depending on charge level or I might be using a regulated supply such as 3.3 V. This low voltage variability is overcome with a MAX756 switching regulator to create a local 5.0 V supply inside the LCD box regardless of host voltage.

The complete sensor controller and DTMF transmitter unit is housed in an IP66 Camden Boss CHDX8-323 enclosure with hinged lid. This is clamped near the top of the rigid aluminium ladder at Pot Sump, which is about 5 m above the typical water level. My Druck sensor has 6 m of cable, just enough to reach the top of the ladder and keep the sensor body underwater. In years past, matchbox-size lumps of wood were left on ladder rungs for months, only some of which floated away, suggesting that the water level never reached anywhere near the top of the ladder.

Receiver Design

The receiver does not need to be micropower because it is located at the surface end of the SWT cable in the Plant room by the cave entrance, which has mains power. It is also in the nature of receivers that they are "always on", thus power-hungry, as compared to transmitters, which can be activated on demand and then returned to sleep for long periods.

The power supply is regulated to 3.3 V internally. The incoming DTMF audio is amplified by an uncommitted op-amp within the HT9170B decoder to a usable voltage level before decoding. The PIC18 firmware converts the DTMF code number obtained (0-15) to ASCII characters and sends them to a PC via USB using the 3.3 V version of an FTDI adaptor cable, part number TTL-232R-3V3-WE. A test version of the receiver can be powered from an alkaline battery pack with USB left unconnected, and this illuminates a four-LED array with the binary equivalent of the incoming DTMF as each 0-15 code is decoded. Another version can generate fake packets within the receiver box and transmit those over USB.

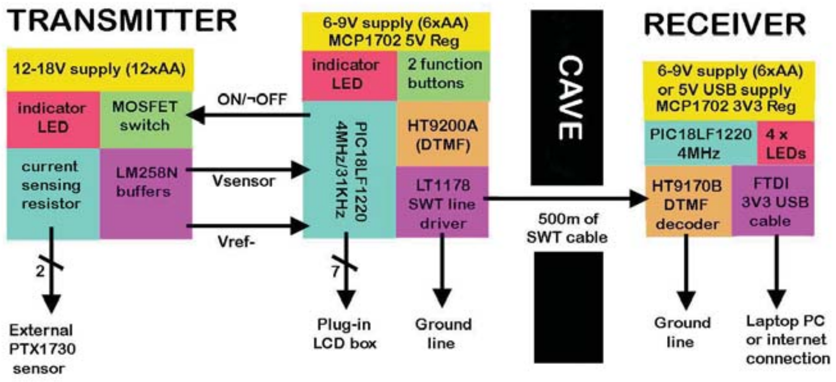


Figure 7 – System Block Diagram

Similarly, the transmitter box at Pot Sump has a comms test version which loops every few seconds sending fake packets rather than real readings from the Druck sensor at 15-minute intervals.

Power Supplies

A complication in the technology described is the different operating voltages of its various subsystems. The Druck submersible sensor requires minimum 12 V DC at its input. The PIC18 controlling that is running at 5 V. Fortunately, the DTMF generator chip (Holtek HT9200A) also works at 5 V as does the circuitry to drive the cave phone line with the generated DTMF audio tones. The LT1178 transmitter op-amp also runs at 5 V, but its output has been lowered to 3.3 V so as to be compatible with the receiver in anticipation of a future Internet connection using some typical 3.3 V telecom module rather than a PC.

Readers should be warned that the 3.3 V FTDI USB cable actually supplies 5.0 V at its bare-wire end, which is derived from a USB-A socket on the connected PC. The "3.3" refers only to the UART TTL voltage level and not to the supply voltage wires within the USB cable!

In a real cave rescue incident, there could be an issue when the SWT line will be driven at 12 V by real SWT phone handsets, thus making it necessary to disconnect the water depth monitoring system from the

Figure 8 – Pot Sump

The transmitter unit near the top of the ladder with the sensor mounted 6m below.

SWT line at both ends. But that does not seem burdensome since cave rescue would in any case have to connect their own equipment at both ends of the phone cable during some incident.

Cloud Connection Options

Sending live data to the Internet to make charts and tables available on a webpage involves using https to invoke server-side code. The next article will look at this in detail. Candidate methods being considered for this project are to forward data via:

1. a mobile telephone network, but the cave entrance itself may be a 'not-spot', thus requiring another link, perhaps a cable which reaches higher ground
2. the Show Cave premises offer wi-fi, but this may not reach as far as the cave entrance
3. LoRa to SWCC cottages, which are 2 km away on almost a line of sight - this link has already been proven by sending fake data in one hop
4. LoRa to the National Park Visitor Centre, which is only 1 km away and on a line of sight from the cave entrance and it definitely provides free continuous wi-fi
5. a Show Cave wireless network being developed by other cavers for their science projects and to assist the Show Cave business, but this is not yet operational.

References

[1] **ogof.org.uk/dan-yr-ogof.html**

[2] **showcaves.co.uk/dan_yr_ogof.html**

[3] **linetop.co.uk/creg/creg_j037.pdf**

[4] **linetop.co.uk/creg/creg_j093.pdf**

[5] **linetop.co.uk/cssdata/swt.htm**

Writing for the Journal

Please let me know if you are working on anything that you feel would be of interest to other readers.

I firmly believe that we should share our ideas and experiences, getting things on the record so that others can learn, develop, and contribute.

We're keen to make the authors' experience as painless as possible and are happy to take scrappy notes and scribbles and format them appropriately (though I'm also happy to provide a template if you'd like to do a trial layout).

Whatever you are working on, please get in contact!

Rob Gill

Field Equations in Conducting Media

*When studying the fields from a small dipole antenna (electric or magnetic) in a conducting medium, it is sometimes helpful to use complete field expressions rather than the simpler quasi-static approximations. It is not always easy to find these in a textbook, so **David Gibson** has set them out here, as a useful reference.*

The fields due to current distributions in space can be derived using the retarded magnetic vector potential (MVP). It is an advanced technique so I won't give any detail here, except to say that the method is fascinating because we would be hard-pressed to explain what MVP actually is – it corresponds to nothing in the physical world, and yet is highly useful in setting up and solving electromagnetic puzzles. For a basic discussion of its bizarre nature, see (Susskind and Hrabovsky, 2014) §11.

In textbooks, you may see the process applied to a small electric dipole. That is usually chosen as an example because the current distribution in an electric dipole is easy to write down.

Magnetic Dipole

For a small loop, the procedure is far more tedious. See (Ramo, Whinnery and Van Duzer, 1984) §3.19ff or (Gibson, 2003) §A2.1. In that reference, I gave a derivation of the electric (**E**) and magnetic (**H**) vector fields from a small magnetic dipole with magnetic moment m_d, in a vacuum. In spherical polar coordinates (r, θ, ϕ), the fields at frequency ω are given by

$$\mathbf{H} = \frac{m_d}{4\pi r^3} e^{-jk_o r} \left\{ \begin{aligned} & 2\cos\theta\left(1 + jk_0 r\right)\hat{\mathbf{r}} + \\ & \sin\theta\left(1 + jk_0 r - (k_0 r)^2\right)\hat{\boldsymbol{\theta}} \end{aligned} \right\}$$

$$\mathbf{E} = \frac{j\omega\mu_0\, m_d}{4\pi r^2} \sin\theta\left(1 + jk_0 r\right) e^{-jk_0 r}\ \hat{\boldsymbol{\varphi}}$$

The assumption of a 'good' conductor and the resulting complex permittivity of the medium leads to a substitution for the wave number $k_0 = 2\pi/\lambda$ as $k_0 \to (1+j)/\delta$, which allows us to write those two equations in terms of the parameter

$$T = r/\delta$$

as shown in the box below. However, strictly speaking, these do not apply to the half-plane model (surface *v.* underground) that we need. See (Gibson, 2010) §2.3.5.

For clarity, I have introduced functions that define the shape of the field lines, with **bar** describing a 'bar magnet' shape, and **circ** describing concentric circles.

Electric Dipole

From a similar starting point, using an electric dipole with moment p_d, we write

$$\mathbf{E} = \frac{p_d / j\omega}{4\pi\varepsilon_0 r^3} e^{-jk_o r} \left\{ \begin{aligned} & 2\cos\theta\left(1 + jk_0 r\right)\hat{\mathbf{r}} + \\ & \sin\theta\left(1 + jk_0 r - (k_0 r)^2\right)\hat{\boldsymbol{\theta}} \end{aligned} \right\}$$

$$\mathbf{H} = \frac{p_d}{4\pi r^2} \sin\theta\left(1 + jk_0 r\right) e^{-jk_0 r}\ \hat{\boldsymbol{\varphi}}$$

We can make the same substitutions as before but, additionally, we note that ε_0, which appears explicitly in the above equation, must also be written in terms of the complex permittivity and that, in a good conductor with $\sigma/\omega\varepsilon >> 1$, this means that $\varepsilon_0 \to \sigma/j\omega$. Finally, we note that, under all conditions, $\mathbf{J} = \sigma\mathbf{E}$, so that the equations are as shown in the box.

Note that the magnetic field from a magnetic dipole and the electric field from an electric dipole both have a $1/r^3$ term and a bar magnet shape, whereas the *electric* field from a magnetic dipole and the *magnetic* field from an electric dipole both feature $1/r^2$ and concentric circles.

Further Reading

Gibson, D. (2010). *Channel Characterisation and System Design for Sub-Surface Communications.* ISBN 978-1-4457-6953-0. **Available at lulu.com/content/5870557.**

Ramo, S., J. R. Whinnery and T. Van Duzer (1984). *Fields and Waves in Communication Electronics.* New York: John Wiley. (2nd Ed.).

Susskind, Leonard & George Hrabovsky (2014), *Classical Mechanics: The Theoretical Minimum,* London: Penguin Books, ISBN 978-0-141-97622-8.

Magnetic Dipole	Electric Dipole	Multiplier	Definition of Functions
$\mathbf{H} = \frac{m_d}{4\pi r^3} \times \ldots$	$\mathbf{J} = \frac{p_d}{4\pi r^3} \times \ldots$	$\mathbf{bar}(\theta, T)\, e^{-jT} e^{-T}$	$\mathbf{bar}(\theta, T) =$ $2\cos\theta\left(1 + (1 + j)T\right)\hat{\mathbf{r}} +$ $\sin\theta\left(1 + (1 + j)T + 2jT^2\right)\hat{\boldsymbol{\theta}}$
$\mathbf{E} = j\omega\mu_0 \frac{m_d}{4\pi r^2} \times \ldots$	$\mathbf{H} = \frac{p_d}{4\pi r^2} \times \ldots$	$\mathbf{circ}(\theta, T)\, e^{-jT} e^{-T}$	$\mathbf{circ}(\theta, T) =$ $\sin\theta\left(1 + (1 + j)T\right)\hat{\boldsymbol{\varphi}}$